Interactive Engineering Lab: Thermodynamic Modeling of Cold-Climate Heat Pump Balance Points, Inverter COP Deratings, and Supplemental Electric Resistance Loads

Courseware & Educational Resource Series — HVACLogic Open Building Science Initiative

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Canonical Engine: `https://hvaclogic.org/calculators/heat-pump-size-calculator`

Companion Hub: `https://hvaclogic.org/heating-systems`

Open Dataset: `https://doi.org/10.6084/m9.figshare.33456928`

Target Level: Mechanical Engineering Undergraduates, Building Science Researchers, and HVAC/R Apprentices (CTE / Post-Secondary)

Abstract

As building electrification accelerates, air-source heat pump (ASHP) systems must provide primary space heating across extreme sub-freezing ambient temperatures. Unlike combustion furnaces whose thermal heat output remains invariant to ambient temperature, the heating capacity and Coefficient of Performance (COP) of vapor-compression heat pumps degrade monotonically as outdoor temperatures fall, driven by reduced refrigerant vapor density entering the compressor and widening compression pressure ratios.

This interactive laboratory exercise guides engineering and technology students through the quantitative analysis of cold-climate inverter-driven heat pump dynamics. Students formulate the building envelope heat loss line, overlay variable-speed compressor heating capacity curves across -15°F to 55°F (-26°C to 13°C), calculate the exact thermodynamic thermal balance point (T_{bal}), evaluate defrost degradation penalties between 30°F and 40°F, and quantify supplemental electric resistance strip sizing ($COP = 1.0$) to avoid winter grid spikes. Numerical simulations are conducted and validated using the deterministic HVACLogic Heat Pump Sizing & Balance Point Engine.

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1. Educational Objectives

Upon completion of this computational laboratory unit, students will be able to: 1. **Formulate Envelope Heat Loss Lines**: Express building space heating load as a linear function of outdoor dry-bulb temperature using design thermal transmission and infiltration parameters. 2. **Model Non-Linear Compressor Derating**: Map inverter heat pump heating capacity $q_{hp}(T_{oa})$ across minimum, intermediate, rated, and maximum compressor speeds. 3. **Solve the Thermodynamic Balance Point**: Calculate the exact outdoor ambient intersection where building thermal envelope demand equals heat pump maximum output capacity ($q_{loss}(T_{bal}) = q_{hp,max}(T_{bal})$). 4. **Quantify Supplemental Electric Deficits**: Calculate required supplemental electric strip heating capacity (kW_{aux}) and evaluate the seasonal energy degradation resulting from undersized or oversized heat pump selection. 5. **Analyze Latent Frosting & Defrost Penalties**: Account for the parasitic energy losses that occur between 30°F and 38°F when coil surface temperatures drop below the atmospheric dew point and freeze, requiring reverse-cycle hot-gas defrost.

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2. Governing Thermodynamic Principles & Mathematical Formulation

2.1 Building Envelope Heating Demand Curve

Space heating load is modeled as a linear heat transfer system proportional to the indoor-outdoor temperature difference:

```
q_loss(T_oa) = (UA_eff + 1.08  CFM_inf)  (T_indoor - T_oa)
```

Where: - T_{indoor} = Indoor setpoint design temperature (standard: 70°F / 21.1°C). - T_{oa} = Outdoor ambient dry-bulb temperature ($^\circ\text{F}$). - UA_{eff} = Effective overall building thermal transmission conductance ($\text{Btu}/(\text{hr}\cdot^\circ\text{F})$). - CFM_{inf} = Natural and mechanical envelope infiltration rate (CFM).

At the 99% winter design temperature (T_{design}), the building exhibits its peak design heating load (q_{design}):

```
Slope_loss = q_loss(T_oa) / (T_indoor - T_oa) = q_design / (T_indoor - T_design)
```

2.2 Heat Pump Heating Capacity Derating Curve

Unlike electric resistance heaters where electrical work converts directly to heat ($COP = 1.00$):

```
q_res = 3,412 * kW_elec
```

A vapor-compression heat pump transfers thermal energy from the low-temperature outdoor environment to the conditioned space with a thermodynamic Coefficient of Performance:

```
COP = q_heating / W_comp
```

Because low outdoor suction temperatures reduce compressor mass flow rate ($\dot{m} = \rho_{suct} \cdot V_{disp} \cdot \eta_v$), available heating output declines with ambient temperature. Modern cold-climate variable-speed inverters utilize enhanced vapor injection (EVI) or hyper-inverter overspeeding to mitigate this loss, modeled as a piecewise second-order polynomial:

```
q_hp(T_oa) = q_rated  [ a_0 + a_1  T_oa + a_2  (T_oa)^2 ]  Factor_defrost(T_oa)
```

Where: - For $T_{oa} \ge 47^\circ\text{F}$: Nominal AHRI Standard 210/240 high-temperature rating condition. - For $17^\circ\text{F} \le T_{oa} < 47^\circ\text{F}$: Intermediate low-temperature condition. - For $T_{oa} < 17^\circ\text{F}$: Cold-climate extreme operation down to -15°F (NEEP ccASHP specification). - $\text{Factor}_{defrost}(T_{oa})$: Defrost degradation factor, exhibiting a minimum (0.85 to 0.88) between 30°F and 35°F due to latent coil frosting.

2.3 Thermal Balance Point (T_{bal})

The thermal balance point is defined as the outdoor dry-bulb temperature where the building envelope heat loss line intersects the maximum heating capacity of the heat pump:

```
q_loss(T_bal) = q_hp,max(T_bal)
```

- **Condition $T_{oa} > T_{bal}$**: The heat pump has surplus heating capacity. Variable-speed systems modulate down to minimum inverter speed to maintain continuous steady-state comfort and maximize seasonal efficiency ($COP > 3.5$). - **Condition $T_{oa} < T_{bal}$**: The building heat loss exceeds the heat pump capacity. Supplemental auxiliary heat (q_{aux}) must engage to maintain indoor design conditions:

```
q_aux(T_oa) = q_loss(T_oa) - q_hp,max(T_oa)
kW_aux = q_aux(T_oa) / 3,412
```

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3. Laboratory Protocol & Simulation Experiment

Experiment Scenario: Cold-Climate Residential Decarbonization

Students analyze a detached single-family home located in Climate Zone 5 (Chicago / Minneapolis border): - Conditioned Floor Area: $2,400\text{ sq ft}$ - Winter 99% Design Outdoor Temperature: -5°F - Indoor Heating Setpoint: 70°F ($\Delta T_{design} = 75^\circ\text{F}$) - Calculated ACCA Manual J Design Heat Loss: $45,000\text{ Btu/hr}$ - Summer 1% Cooling Load: $32,000\text{ Btu/hr}$ (2.67 Tons)

Step-by-Step Simulation Procedure

1. Navigate to the [HVACLogic Heat Pump Sizing & Balance Point Engine]. 2. Input the design heat loss ($45,000\text{ Btu/hr}$) and design outdoor temperature (-5°F). 3. Evaluate three equipment selection scenarios: - **Scenario A (Sized strictly on summer cooling load)**: 2.5-Ton Heat Pump ($30,000\text{ Btu/hr}$ nominal). - **Scenario B (Cold-climate inverter sized on heating balance)**: 3.5-Ton Cold-Climate Inverter with 80% capacity retention at -5°F. - **Scenario C (Oversized non-inverter unit)**: 5.0-Ton Single-Stage Heat Pump ($60,000\text{ Btu/hr}$ nominal). 4. Record the calculated balance points, auxiliary heat strip kilowatt requirements, and low-temperature COP values in Table 1.

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4. Benchmark Verification Table

Parameter	Scenario A (2.5-Ton Cooling Match)	Scenario B (3.5-Ton Cold-Climate Inverter)	Scenario C (5.0-Ton Single-Stage)
Rated Heating Capacity (47°F)	28,000 Btu/hr	42,000 Btu/hr	58,000 Btu/hr
Capacity at 17°F	17,500 Btu/hr	34,500 Btu/hr	36,000 Btu/hr
Capacity at -5°F Design	9,800 Btu/hr	33,600 Btu/hr	21,000 Btu/hr
Thermal Balance Point (T_{bal})	**36°F (2.2°C)**	**14°F (-10.0°C)**	**11°F (-11.7°C)**
Deficit at -5°F Design	35,200 Btu/hr	11,400 Btu/hr	24,000 Btu/hr
Required Aux Resistance Strip	**10.3 kW**	**3.3 kW**	**7.0 kW**
Summer Modulation Capability	Minimum 100% (No modulation)	Modulates down to 25% (8,000 Btu/hr)	Severe short cycling (< 6 min run)

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5. Pedagogical Discussion Questions for Students

1. **The Cooling Load vs. Heating Balance Paradox**: Why does sizing a heat pump solely on Manual S summer cooling capacity in heating-dominated climates cause excessive winter utility bills? 2. **Defrost Dynamics**: Explain why the lowest operating COP of an air-source heat pump often occurs around 33°F to 35°F rather than

15°F when coil frosting triggers frequent reverse-cycle defrost sequences. 3. **Grid Infrastructure Impact**: How does lowering the thermal balance point from 36°F to 14°F protect municipal electrical substations during polar vortex events?

References & Interactive Resources

1. **HVACLogic Engineering Suite**: Cold-Climate Heat Pump Sizing & Balance Point Calculator 2. **Companion Envelope Modeling**: ACCA Manual J Building Heat Loss Calculator 3. **Air-Conditioning, Heating, and Refrigeration Institute**: *AHRI Standard 210/240: Performance Rating of Unitary Air-Conditioning & Air-Source Heat Pump Equipment*, 2023. 4. **Northeast Energy Efficiency Partnerships (NEEP)**: *Cold Climate Air-Source Heat Pump Specification Version 4.0*, 2024. 5. **ASHRAE**: *Handbook of Fundamentals*, Chapter 18: Non-Residential Cooling and Heating Load Calculations, 2025.