Thermodynamic Formulations of ASHRAE Hyland-Wexler Moist Air Psychrometrics and Numerical Energy-Balance Solvers for Building Sizing and Field Diagnostics

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ABSTRACT

Accurate evaluation of moist air thermodynamic properties is fundamental to building energy simulation, Sensible Heat Ratio (SHR) load splitting, and HVAC equipment sizing. Simplified quadratic approximations or linear psychrometric shortcuts introduce significant cumulative errors (often exceeding 5% to 8%) when applied across non-standard barometric elevations and elevated moisture contents. This monograph details the mathematical implementation of governing ASHRAE Fundamentals (Chapter 1) Hyland-Wexler formulations for saturation vapor pressure across liquid water (32°F to 392°F) and sub-freezing ice (-148°F to 32°F). We establish the numerical inversion framework for dew-point determination via logarithmic pressure polynomials and derive a 1D numerical energy-balance bisection solver for wet-bulb equilibrium convergence without closed-form algebraic solutions. Complete algorithmic architectures for humidity ratio (W), grains of moisture per pound of dry air, specific enthalpy (h), specific volume (v), and moist air density (ρ) are validated against experimental thermodynamic tables. Interactive open-access computational implementations and validation benchmarks are integrated into the HVACLogic Deterministic Building Science Suite (https://hvaclogic.org/calculators/psychrometric-calculator).

Keywords: Psychrometrics, Moist Air Thermodynamics, ASHRAE Hyland-Wexler, Enthalpy Sizing, Wet Bulb Convergence, Sensible Heat Ratio, Decarbonization, Zero-Database Engineering.

1. Introduction & Fundamental Governing Laws

Atmospheric moist air is treated thermodynamically as a binary mixture composed of dry air and water vapor. Under Dalton's Law of Partial Pressures, the barometric total pressure P_{atm} equals the sum of the partial pressures exerted independently by dry air (P_{da}) and water vapor (P_w):

$$P_{atm} = P_{da} + P_w$$

At standard sea level, P_{atm} is defined as 14.696 psia (101.325 kPa). To account for altitudinal pressure gradients, ASHRAE specifies the barometric decay formula for elevations up to 15,000 ft:

$$P_{atm} = 14.696 \times (1 - 6.8754\times10^{-6} \times h)^{5.2559}$$

where h is elevation above sea level in feet. Neglecting barometric altitude corrections leads to substantial density and mass-flow calculation errors in high-altitude zones (e.g., Denver, CO at 5,280 ft, where P_{atm} drops to 12.15 psia).

2. Governing Saturation Pressure: The Hyland-Wexler Formulations

The saturation vapor pressure P_{ws} defines the equilibrium vapor pressure exerted by moist air at full saturation. ASHRAE endorses the Hyland and Wexler (1983) formulations evaluated over absolute thermodynamic temperature T in degrees Rankine ($T_R = T_{°F} + 459.67$).

2.1 Saturation Over Liquid Water (32°F to 392°F)

$$ln(P_{ws}) = C_8/T + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13}ln(T)$$

where empirical coefficients are:

- $C_8 = -1.0440397 \times 10^4$
- $C_9 = -1.1294650 \times 10^1$
- $C_{10} = -2.7022355 \times 10^{-2}$

- $C_{11} = 1.2890360 \times 10^{-5}$
- $C_{12} = -2.4780681 \times 10^{-9}$
- $C_{13} = 6.5459673$

2.2 Saturation Over Ice (−148°F to 32°F)

$$ln(P_{ws}) = C_1/T + C_2 + C_3T + C_4T^2 + C_5T^3 + C_6T^4 + C_7ln(T)$$

where coefficients over the ice phase boundary are:

- $C_1 = -1.0214165 \times 10^4$
- $C_2 = -4.8932428$
- $C_3 = -5.3765794 \times 10^{-3}$
- $C_4 = 1.9202377 \times 10^{-7}$
- $C_5 = 3.5575832 \times 10^{-10}$
- $C_6 = -9.0344688 \times 10^{-14}$
- $C_7 = 4.1635019$

3. Humidity Ratio and Grains of Moisture

The humidity ratio W represents the dimensionless mass ratio of water vapor per pound of dry air:

$$W = 0.621945 \times [\, P_w / (P_{atm} - P_w) \,]$$

The constant 0.621945 corresponds to the ratio of molecular masses M_w/M_{da} = 18.01528 / 28.966. In building science, moisture is tracked in grains per pound (gr/lb), where 1 lb = 7,000 grains:

$$W_{grains} = W \times 7{,}000$$

Under standard indoor comfort conditions (75°F dry-bulb, 50% relative humidity), moisture content stands at ~65 gr/lb. Exceeding 70 gr/lb exponentially elevates dust mite and mold spore proliferation.

4. Dew Point Determination via Inverse Logarithmic Polynomials

The dew point temperature T_{dp} is the temperature at which condensation begins at constant pressure. ASHRAE provides an explicit inverse formulation based on $\alpha = \ln(P_w)$:

$$T_{dp} = 100.45 + 33.193\alpha + 2.319\alpha^2 + 0.17074\alpha^3 + 1.2063(P_w)^{0.1984}$$

valid for $T_{dp} \geq 32$°F ($P_w \geq 0.08865$ psia).

5. Wet Bulb Equilibrium via Numerical Energy Balance

Wet bulb temperature T_{wb} represents dynamic equilibrium where sensible convective heat transfer from air balances evaporative mass transfer from a wetted wick. The governing thermodynamic energy balance is:

$$W = [\, (1093 - 0.556\, T_{wb})W_{s,wb} - 0.24(T_{db} - T_{wb}) \,] / [\, 1093 + 0.444\, T_{db} - T_{wb} \,]$$

Because $W_{s,wb}$ is strongly non-linear in T_{wb}, this formulation cannot be solved in closed algebraic form. HVACLogic implements an ultra-fast bisection root-finder bounded within [−20°F, T_{db}] that converges to within 0.01°F in under 25 iterations.

6. Volumetric & Enthalpy Formulations

6.1 Specific Enthalpy (h)

$$h = 0.24\, T_{db} + W \times (1061 + 0.444\, T_{db}) \quad [BTU/lb]$$

6.2 Specific Volume (v) & Density (ρ)

$$v = [\, 53.352 \times T_R \times (1 + 1.607858\, W) \,] / (P_{atm} \times 144)$$

$$\rho = (1 + W) / v \quad [lb/ft^3]$$

7. Experimental Validation Matrix

Below is the empirical benchmark matrix solved by the HVACLogic deterministic engine compared against ASHRAE Fundamentals Chapter 1 standard tables:

Condition	T_{db} (°F)	RH (%)	T_{wb} (°F)	T_{dp} (°F)	W (gr/lb)	h (BTU/lb)
Standard Comfort	75.0	50.0	62.5	55.1	64.9	28.14
Cooling Coil Exit	55.0	90.0	53.6	52.2	58.1	22.42
Peak Summer	95.0	40.0	75.2	67.2	98.4	38.61
Winter Design	20.0	60.0	18.1	10.3	9.1	5.88

8. Architectural Implementation & Conclusion

By implementing these exact formulations in pure, side-effect-free TypeScript, HVACLogic provides sub-millisecond psychrometric state evaluation directly within client browsers. This eliminates server latency, protects user project confidentiality, and provides engineering-grade accuracy across modern building electrification workflows.

References

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